

## 3D HIGHER ORDER MODELING IN THE BEM/FEM HYBRID FORMULATION

P. W. Fink\*  
NASA-JSC  
Houston, TX 77058

D. R. Wilton  
University of Houston  
Houston, TX 77204-4793

Higher order divergence- and curl-conforming bases have been shown to provide significant benefits, in both convergence rate and accuracy, in the 2D hybrid finite element/boundary element formulation (P. Fink and D. Wilton, National Radio Science Meeting, Boulder, CO, Jan. 2000). A critical issue in achieving the potential for accuracy of the approach is the accurate evaluation of all matrix elements. These involve products of high order polynomials and, in some instances, singular Green's functions. In the 2D formulation, the use of a generalized Gaussian quadrature method was found to greatly facilitate the computation and to improve the accuracy of the boundary integral equation self-terms. In this paper, a 3D, hybrid electric field formulation employing higher order bases and higher order elements is presented. The improvements in convergence rate and accuracy, compared to those resulting from lower order modeling, are established. Techniques developed to facilitate the computation of the boundary integral self-terms are also shown to improve the accuracy of these terms. Finally, simple preconditioning techniques are used in conjunction with iterative solution procedures to solve the resulting linear system efficiently.

In order to handle the boundary integral singularities in the 3D formulation, the parent element—either a triangle or rectangle—is subdivided into a set of sub-triangles with a common vertex at the singularity. The contribution to the integral from each of the sub-triangles is computed using the Duffy transformation to remove the singularity. This method is shown to greatly facilitate the self-term computation when the bases are of higher order. In addition, the sub-triangles can be further divided to achieve near arbitrary accuracy in the self-term computation. An efficient method for subdividing the parent element is presented.

The accuracy obtained using higher order bases is compared to that obtained using lower order bases when the number of unknowns is approximately equal. Also, convergence rates obtained using higher order bases are compared to those obtained with lower order bases for selected sample problems. The convergence rates of the iterative procedure are compared for different preconditioners.